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**EFFECT OF MAGNETIC FIELD ON SUPERCONDUCTING COMPLEX RESISTANCE**  
**ACCORDING TO QUANTUM MECHANICS**

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**ABSTRACT**

The expression of quantum resistance is used to express it as real and imaginary part in the presence of external magnetic field. The real part stands for superconducting resistance. The model indicates that when the external magnetic strength exceeds certain critical value superconductivity is destroyed because the resistance does not vanish.

**Keywords-** *Critical temperature superconductivity, zero resistance, plasma Equation.*

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**I. INTRODUCTION**

Superconductors are distinguished from perfect conductors by their total exclusion of magnetic fields [1]. In an external magnetic field, magnetic flux penetrates type-II superconductors via vortices, each carrying one flux quantum. The vortices form lattices of resistive material embedded in the non-resistive superconductor and can reveal the nature of the ground state.

The phenomenon of superconductivity, in which the electrical resistance of certain materials completely vanishes at low temperatures, is one of the most interesting and sophisticated in condensed matter physics. Also The phenomenon of superconductivity has always been very exciting, both for its fundamental scientific interest and because of its many applications.

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There are two types of superconductors, I and II, characterized by the behavior in an applied magnetic field [2,3]. Type I superconductors depends on Critical Temperature  $T_c$  and Critical Magnetic Field  $B_c$ .

In the presence of an applied magnetic field  $B$ , the value of  $T_c$  decreases with increasing magnetic field for several type I superconductors. When the magnetic field exceeds the critical field,  $B_c$ , the superconducting state is destroyed and the material behaves as a normal conductor with finite resistance. Type II also depends on two critical magnetic fields, designated  $B_{c1}$  and  $B_{c2}$ . When the external magnetic field is less than the lower critical field  $B_{c1}$ , the material is entirely superconducting and there is no flux penetration, just as with type I superconductors. When the external field exceeds the upper critical field  $B_{c2}$ , the flux penetrates completely and the superconducting state is destroyed, just as for type I materials. For fields lying between  $B_{c1}$  and  $B_{c2}$ , however, the material is in a mixed state, referred to as the vortex state [3-5].

Thus plasma equation is suitable for describing behavior of bulk matter [6-8]. Thus it can be used to develop quantum equation for particles moving inside a certain medium [9]. Such equation can reduce quantum equation from large degrees of free dimension to 3dimensionson space only. Such equation was first developed by M.Dirar and Rasha. A [10]. This equation is used to explain some Schrödinger behavior, unfortunately this approach is complex mathematically. Thus there is a need for a simple model that can explain some Schrödinger phenomena. Section (2) is devoted for quantum equation derived from energy equation found from plasma equation. The solution and equation expression for resistance is exhibited in section (3). Section (4) is concerned with finding critical temperature and quantum resistance . Discussion and conclusion are in section (5) and (6) respectively.

**II. COMPLEX QUANTUM RESISTANCE MODEL**

Plasma equation describes particles having specific charge . This equation describes the electron motion easily. This is since electrons are charged. For pressure exerted by the gas plasma equation becomes:

$$mn \frac{dv}{dt} = -\nabla P + F \tag{1}$$

But for pressure exerted by the medium on the electron gas, the equation become:

$$mn \frac{dv}{dt} = \nabla P + F = \nabla P - \nabla V \tag{2}$$

In one dimensions, the equation becomes:

$$mn \frac{dv}{dx} \frac{dx}{dt} = \frac{d(nkT)}{dx} - \frac{dnv}{dx}$$

$$mn \frac{v dv}{dx} = \frac{d}{dx} [nkT - nv]$$

where V is the potential for one particle

$$mn \frac{d1/2 v^2}{dx} = \frac{d}{dx} [nkT - nV]$$

Thus in integrating both sides by assuming n to be constant, or in- dependent of k, yields:

$$\frac{n}{2} m v^2 = nkT - nV + c$$

$$\frac{1}{2} m v^2 + V - kT = \frac{c}{n} = \text{constant} = E$$

This constant of motion stands for energy, thus:

$$E = \frac{P^2}{2m} + V - kT \tag{3}$$

Multiplying by  $\psi$ , yields:

$$E\psi = \frac{P^2}{2m} \psi + V\psi - kT\psi \tag{4}$$

According to the wave nature of particles:

$$\psi = A e^{\frac{i}{\hbar}(px - Et)}$$

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi$$

$$-\hbar^2 \nabla^2 \psi = P^2 \psi \tag{5}$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi - kT\psi \tag{6}$$

The time in dependent equation becomes:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi - kT\psi = E\psi \tag{7}$$

Consider the case when these electrons wave subjected to constant crystal field  $V_0$ . This assumption is quite natural as far as particles are distributed homogenously. Thus, equation (7) becomes:

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V_0 \psi - kT \psi = E \psi \tag{8}$$

One can suggest the solution to be:

$$\psi = A e^{ikx} \tag{9}$$

A direct substitution yields:

$$\left( \frac{\hbar^2}{2m} k^2 + V_0 - kT \right) \psi = E \psi$$

Therefore:

$$K = \frac{\sqrt{2m(E+kT-V_0)}}{\hbar} \tag{10}$$

This wave number  $K$  is related to the momentum according to the relation :

$$P = mv = \hbar K = \sqrt{2m(E + kT - V_0)} \tag{11}$$

This relation can be used to find the quantum resistance  $R$  of a certain material . According to classical laws :

$$R = \frac{V}{I} \tag{12}$$

For harmonic oscillator:  $T = V$

For electrons accelerated by the potential . The work done is related to the potential  $V$  and kinetic energy  $K$  according to the relation :

$$W = V = \frac{1}{2} m v^2 \tag{13}$$

But since the current  $I$  is gives by .

$$I = nevA \tag{14}$$

$$R = \frac{mv^2}{2nevA} = \frac{mv}{2n e A} = \frac{P}{2n e A} \tag{15}$$

From (12) and (13) :

$$R = \frac{\sqrt{2m(E+kT-V_0)}}{2neA} \tag{16}$$

Splitting  $R$  to real part  $R_r$  and imaginary part  $R_i$  :

$$R = R_r + jR_i \tag{17}$$

According to equation (16)  $R$  becomes pure imaginary , when :

$$E + KT - V_0 < 0$$

$$KT < V_0 - E$$

$$T < \frac{(V_0 - E)}{K} \tag{18}$$

Thus the critical temperature is given by :

$$T_c = \frac{V_0 - E}{K}$$

Thus equation (16) becomes

$$R = \frac{\sqrt{2mK(T - T_c)}}{2enA} \tag{19}$$

Which requires:

$$V_0 > E \text{ for } T < T_c$$

Thus  $R$  becomes

$$R = \frac{j\sqrt{2mK(T - T_c)}}{2enA} \tag{20}$$

$$R = jR_i \tag{21}$$

Using equation (16) yields

$$R_s = 0 \tag{22}$$

Thus the superconductivity resistance vanishes for all  $T$  less than the critical value.

When an external magnetic field of flux density  $B$  is applied, the total the total medium field is given by

$$B_m = B - B_i \tag{23}$$

Where  $B_i$  is the internal flux density. The corresponding potential applied on electrons or charges is given by  $V_m$ , thus the total potential in equation (7) becomes

$$V = V_0 \pm V_m \tag{24}$$

$$V_m = m_L \left( \frac{e\hbar}{2m} \right) B_m = C_0^{-1} B_m$$

When the net magnetic potential opposes the crystal field

$$V = V_0 - V_m = V_0 - C_0^{-1} B_m \tag{25}$$

In this case one can rewrite the expression of  $R$  in equation (16) to be

$$R = \frac{\sqrt{2mK(E + KT - V_0 + V_m)}}{2enA} = \frac{\sqrt{2mK(T - T_c + T_m)}}{2enA}$$

$$R = \frac{\sqrt{2mK(T + T_m - T_c)}}{2enA} \quad (26)$$

Consider now the case when  $T_m$  is greater than  $T_c$ , i.e when

$$T_m \geq T_c \quad (27)$$

According to equations (23), (25) and (27) this critical value is given by

$$B_c = C_0(2mKT_c) + B_{ic} \quad (28)$$

In this case the term under the square root is positive always. This means that, it

$$R = R_s + jR_i \quad (29)$$

$$R_i = 0 \quad R_s \neq 0 \quad (20)$$

This means that the superconductivity is destroyed when applying an external magnetic field having strength to satisfy equation (1).

### III. DISCUSSION

In the work done by Dirar and Einas ;quantum mechanical resistance is considered as a sum of real superconducting part and imaginary part as equation (17) shows. In this work the effect of subjecting superconducting to magnetic field is investigated the magnetic field inside superconducting is considered as consisting of external and internal field see equation (23). This magnetic field added to  $B$  an additional medium potential  $V_m$  and a corresponding temperature  $T_m$ , as written in equation (25, 26). Equation (29) shows that when an external magnetic field exceeds a certain critical valve given by (28), the superconducting state is destroyed since  $R_s \neq 0$  As equation (30) indicates.

### IV. CONCLUSION

The complex resistance quantum model, based on temperature dependent Schrodinger equation can easily explain the sc destroy due to the presence of external magnetic field. This means that this model needs to be promoted and applied to describe fully sc state.

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